Connection between stress state and plastic strain increments determined by a computer method

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The ratios of the plastic shear strain and tensile strain to the resolved shear strain are determined on the basis of earlier results obtained by computer for ⟨1 0 0⟩ and ⟨1 1 1⟩ textures and nontextured polycrystals in the case of simultaneous shear and extension. It is shown that the plastic deformation increment vector determined by the application of the plastic work equation fulfils the normality condition prescribed by the generalized flow law.

1. Introduction

It is known that the convexity of the yield surface is required by the Drucker's deformation stability condition [1]. On the basis of the generalized flow law the increment, $d\epsilon_{ij}^{p}$ of the plastic deformation vector is perpendicular to the yield surface that is [2]:

$$d\epsilon_{ij}^{p} = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (1)$$

where f is the yield function and $d\lambda$ is a factor depending on the stress state and the deformation.

In previous papers the yield function of textured and nontextured polycrystals have been determined for the stress state of simultaneous shear and extension [3, 4]. The yield functions obtained obey the condition of convexity. In the present paper the validity of the normality condition is studied and the connections between shear strains, tensile strains and the resolved shear strain are determined.

2. Calculation of the yield function by the computer method

In this section the results of our yield function calculations are summarized [3, 4].

By the method applied the shear stresses of the possible 12 slip systems were determined in all the grains of the polycrystal and the arithmetical mean of the five largest shear stresses was calculated. The condition of plastic flow was regarded to be fulfilled when the arithmetical mean of these shear stresses of the individual grains attained the value of the critical shear stress, $\tau_{\mathrm{r}}^{\mathrm{c}}$. The validity of this averaging method can be explained by the development of internal stresses in the course of plastic deformation [4]. The calculations were made for the stress state of simultaneous shear and extension. The numerical quantity $\tau_{\mathrm{r}}^{\mathrm{c}}/\tau$ was determined as a function of σ/τ, where σ is the tensile stress and τ is the shear stress applied.

The yield function of three kinds of polycrystalline materials of the ⟨1 0 0⟩ and ⟨1 1 1⟩ textured and of the nontextured one was determined. As an example Fig. 1 shows the $\tau_{\mathrm{r}}^{\mathrm{c}}/\tau - \sigma/\tau$ curve for randomly oriented polycrystalline material. This curve can be approached by a hyperbola with eccentricity $e = 3.142$, nearly equal to π. The minor and major axes indicated on Fig. 1 are $a = 0.6154$, $b = 1.833$. By these parameters the computed function can be well approximated by the expression:

$$\frac{\tau_{\mathrm{r}}^{\mathrm{c}}}{\tau} \cong \left[a^2 + \left(\frac{a\sigma}{b\tau}\right)^2\right]^{1/2} \quad (2)$$

From this expression the critical shear stress is:

$$\frac{\tau_{\mathrm{r}}^{\mathrm{c}}}{a} = \left[\tau^2 + \frac{\sigma^2}{b^2}\right]^{1/2} \quad (3)$$

If $\sigma = 0$, then from this expression the yield stress of pure shear is $\tau_{\mathrm{f}} = \tau_{\mathrm{r}}^{\mathrm{c}}/a$, which gives the Taylor

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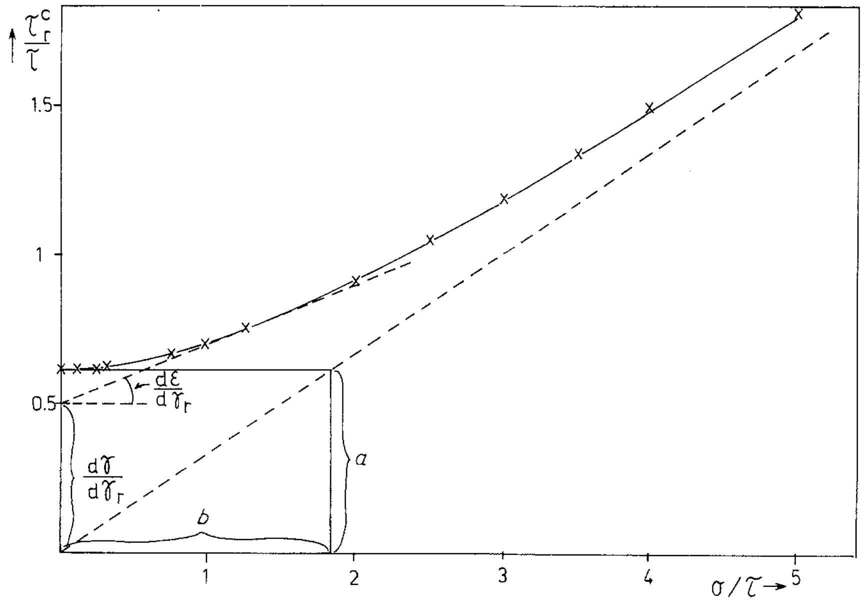

Figure 1 The quantity τ_r^c/τ as a function of σ/τ determined by computer for nontextured polycrystals. (The computed points are denoted by crosses.) The continuous line is a hyperbola.

factor of pure shear $M_\tau = 1/a = 1.625$. Similarly, in the case of $\tau = 0$ the yield stress of pure extension is $\sigma_f = b\tau_r^c/a$, which gives the Taylor factor of pure extension to be $M_\sigma = b/a = 2.978$. By these Taylor factors the connection between the yield stress of pure shear and of pure extension can be obtained:

$$\sigma_f = 1.833\,\tau_f \tag{4}$$

These values are in good agreement with the results known in the literature and obtained in other ways [5–7].

On the basis of Equation 3 the yield function is:

$$f = \frac{\sigma^2}{b^2} + \tau^2 - \tau_f^2 \tag{5}$$

The condition of plastic flow is $f = 0$. On the basis of this condition the connection between the variables σ and τ can be characterized by such an ellipse which placed between the curves obtained from the yield functions due to the von Mises and Tresca criterions (Fig. 2). It is worth mentioning that in the cases of the Mises condition $a = 1/3^{1/2}$, $b = 3^{1/2}$.

The yield functions of ⟨1 0 0⟩ and ⟨1 1 1⟩ textured polycrystals cannot be characterized by ellipses (Figs. 3 and 4). The yield function is linear, if $\sigma/\tau > 2$ and $\sigma/\tau > 5.64$ for the ⟨1 0 0⟩ and ⟨1 1 1⟩ texture, respectively.

For the ⟨1 0 0⟩ texture [3]:

$$\frac{\sigma_f}{\tau_f} = 1.054 \quad M_\sigma = 2.45 \quad M_\tau = 2.33$$

In the case of the ⟨1 1 1⟩ texture:

$$\frac{\sigma_f}{\tau_f} = 2.558 \quad M_\sigma = 3.674 \quad M_\tau = 1.436$$

3. Discussion

To investigate the normality condition given by Equation 1 the plastic tensile and shear deformation increments $d\epsilon$, $d\gamma$ belonging to a given stress state must be calculated. To determine these quantities let us start from the equality of plastic works:

$$\tau_r^c d\gamma_r = \sigma d\epsilon + \tau d\gamma \tag{6}$$

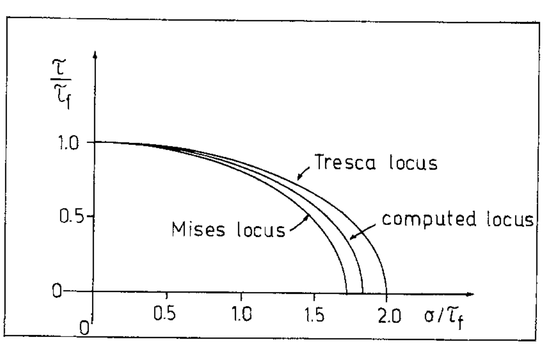

Figure 2 The computed yield function of nontextured polycrystals and the yield functions obtained by the Mises and Tresca criterions for the case of simultaneous shear and extension.

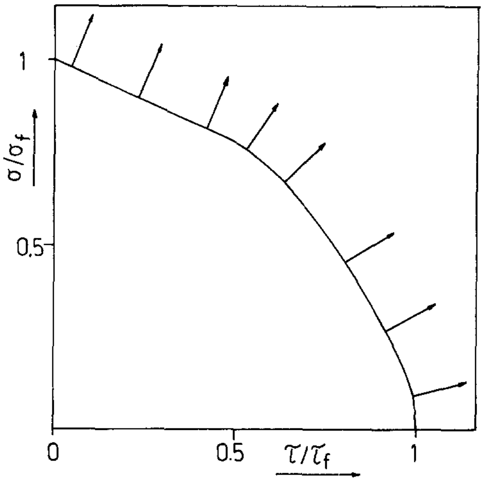

Figure 3 The normalized yield function of the ⟨1 0 0⟩ texture. Arrows denote the direction of plastic deformation increment vectors belonging to the corresponding stress states. The yield function is linear for the stress state $\sigma/\tau > 2$.

where $d\gamma_r$ is the resolved shear strain. With a little change we get:

$$\frac{\tau_r^c}{\tau} = \frac{\sigma}{\tau}\frac{d\epsilon}{d\gamma_r} + \frac{d\gamma}{d\gamma_r} \tag{7}$$

It can be seen from this equation that for a given stress state (σ, τ) the deformation increment ratios $d\epsilon/d\gamma_r$ and $d\gamma/d\gamma_r$ are equal to the slope and to the intersection of the tangent drawn to the $\tau_r^c/\tau - \sigma/\tau$ curve, respectively (Fig. 1). On the

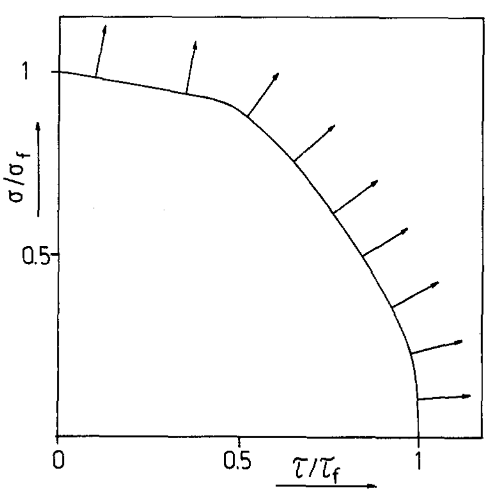

Figure 4 The normalized yield function of the ⟨1 1 1⟩ texture. Arrows denote the direction of plastic deformation increment vectors belonging to the corresponding stress states. The yield function is linear for the stress state $\sigma/\tau > 5.64$.

basis of this interpretation by applying Equation 2 we get the following connections:

$$\frac{d\epsilon}{d\gamma_r} = \frac{a}{b^2\tau_f}\sigma \tag{8a}$$

$$\frac{d\gamma}{d\gamma_r} = \frac{a}{\tau_f}\tau \tag{8b}$$

These equations show that the ratios of the deformation increments are proportional to the corresponding stress. From the connections (Equations 8a and 8b) the ratio of the plastic tensile and plastic shear strains is

$$\frac{d\epsilon}{d\gamma} = \frac{\sigma}{b^2\tau} \tag{9}$$

If we apply the generalized flow law for the yield function given by Equation 4 then we again arrive at Equation 9. This means, that the plastic deformation increment vector $(d\gamma, d\epsilon)$ is perpendicular to the yield surface if the corresponding axes of the stress and strain increment coordinate systems are parallel to each other. Therefore the plastic deformation increment vector obtained by applying Equation 6 fulfils the normality condition prescribed by Equation 1.

It can be shown in a simple way that the deformation increments derived from the generalized flow law are equivalent in general to the ones derived from the plastic work equation, if the critical resolved shear stress can be written in the form

$$\tau_r^c = \tau g\left(\frac{\sigma}{\tau}\right) \tag{10}$$

Using the connection $\tau_f = M_t\tau_r^c$ the yield function in this case is

$$f = M_\tau^2\tau^2 g^2\left(\frac{\sigma}{\tau}\right) - \tau_f^2 \tag{11}$$

In the case of simultaneous shear and extension the critical resolved shear stress can always be given in the form of Expression 10 [3], therefore the flow law is always fulfilled automatically for this case.

The validity of this statement can be seen well in the case of the ⟨1 0 0⟩ and ⟨1 1 1⟩ textured materials too. In Figs. 3 and 4 the vectors drawn to the contour of the yield surface show the plastic deformation increment vectors normal to the yield surface. The vectors were determined graphically by the tangents drawn to the corresponding

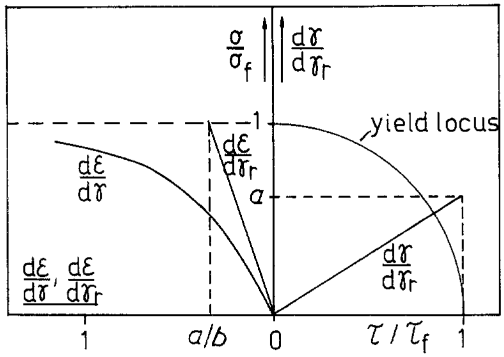

Figure 5 The deformation ratios of nontextured polycrystals as a function of stress state.

$\tau_r^c/\tau - \sigma/\tau$ curve on the basis of Equation 7. The characteristics of these two textures is that along the plane part of the yield surface (along the linear part of the contour) all the three deformation increment ratios $d\epsilon/d\gamma$, $d\epsilon/d\gamma_r$ and $d\gamma/d\gamma_r$ are constants. These ratios and the yield functions for the three kinds of polycrystalline materials can be seen in Figs. 5 to 7. From these figures the ratios of the tensile and shear strains to the resolved shear strain can be obtained for any stress state. These ratios for the stress states $\sigma = 0$ and $\tau = 0$ give the reciprocal values of the Taylor factor for pure shear and for pure extension, respectively.

For nontextured materials in the combined stress state the Taylor factors of pure shear and pure extension characterize connections between an effective tensile or shear strain and the resolved shear strain. Such a connection can be obtained using Equations 2 and 8:

$$\left[\left(\frac{d\epsilon}{d\gamma_r}\right)^2 + \frac{1}{b^2}\left(\frac{d\gamma}{d\gamma_r}\right)^2\right]^{1/2} = \frac{a}{b} \qquad (12)$$

where $b = M_\sigma/M_\tau$ and $a/b = 1/M_\sigma$. It is clear that

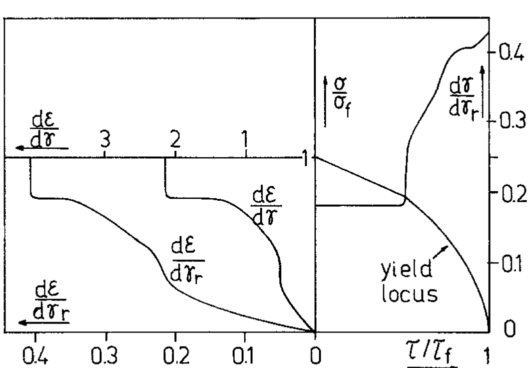

Figure 6 The deformation ratios of the ⟨1 0 0⟩ texture as a function of stress state.

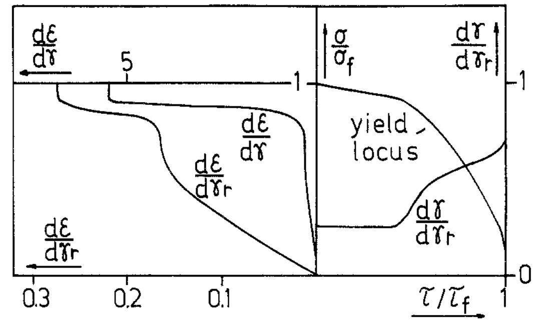

Figure 7 The deformation ratios of the ⟨1 1 1⟩ texture as a function of stress state.

from this equation an equivalent tensile strain $\overline{d\epsilon}$ can be defined by

$$\overline{d\epsilon} = \left[d\epsilon^2 + \frac{1}{b^2}\,d\gamma^2\right]^{1/2}$$

On the basis of Equation 12 an equivalence relation between the pure shear strain (when $d\epsilon = 0$) and pure tensile strain (when $d\gamma = 0$) can also be obtained if $d\gamma_r$ is the same in the two cases. This relation is $d\gamma_p = d\epsilon_p b$, that is $d\epsilon_p$ pure tensile deformation is equivalent to $d\gamma_p/b$ pure shear strain, or $d\gamma_p$ pure shear strain is equivalent to $d\gamma_p/b$ pure tensile strain.

According to Expression 12 the connection between the relative shear and tensile deformation increments can be characterized by a circle with unit radius. Its equation is:

$$\left(\frac{1}{a}\,\frac{d\gamma}{d\gamma_r}\right)^2 + \left(\frac{b}{a}\,\frac{d\epsilon}{d\gamma_r}\right)^2 = 1 \qquad (13)$$

Similarly, the connection between the shear and tensile stresses for plastic yielding can be obtained from Equation 3 as:

$$\left(\frac{\tau}{\tau_f}\right)^2 + \left(\frac{\sigma}{b\tau_f}\right)^2 = 1 \qquad (14)$$

Representing these two circles in the same coordinate system two parallel unit vectors can be defined from which the first one

$$\mathbf{n}_\sigma = \left(\frac{\tau}{\tau_f}, \frac{\sigma}{b\tau_f}\right) \qquad (15)$$

characterizes the stress state, and the second one

$$\mathbf{n}_\epsilon = \left(\frac{1}{a}\,\frac{d\gamma}{d\gamma_r}, \frac{b}{a}\,\frac{d\epsilon}{d\gamma_r}\right) \qquad (16)$$

characterizes the corresponding deformation increments (Fig. 8).

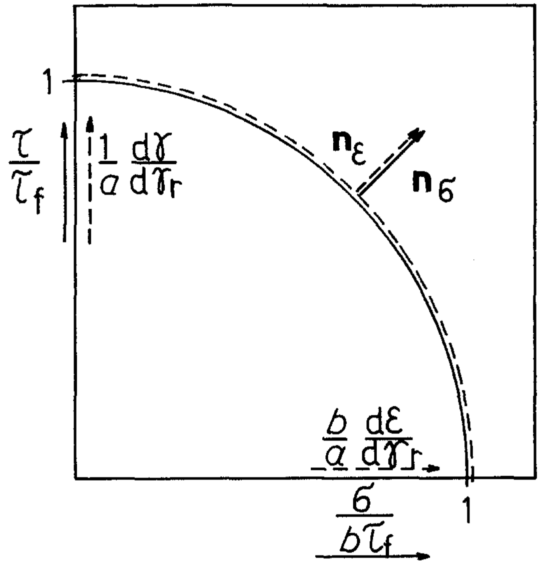

Figure 8 Connection characterizes the stress state and the deformation increments.

It is clear, that both of them are normal to the yield surface. The scalar product of these two vectors gives a relation for the plastic work, because

$$\mathbf{n}_\sigma \mathbf{n}_\epsilon = \frac{1}{a}\frac{\tau}{\tau_f}\frac{d\gamma}{d\gamma_r} + \frac{1}{a}\frac{\sigma}{\tau_f}\frac{d\epsilon}{d\gamma_r} = 1$$

which is just Equation 6 with $a\tau_f = \tau_r^c$.

On the basis of Equation 13 the total resolved shear strain is

$$d\gamma_r = \frac{d\gamma}{a}\left[1 + b^2\left(\frac{d\epsilon}{d\gamma}\right)^2\right]^{1/2} \qquad (17)$$

The quantity $d\epsilon/d\gamma$ can be well measured, for example, in simultaneous torsion and extension experiments so the quantity $d\gamma_r$ can be determined. If we suppose that no change in the orientation distribution of the grains takes place for a finite deformation then we obtain:

$$\gamma_r = \frac{1}{a}\int_0^{\gamma}\left[1 + b^2\left(\frac{d\epsilon}{d\gamma}\right)^2\right]^{1/2} d\gamma$$

Measuring the quantity τ_f as a function of deformation and using τ_r^c and γ_r a single τ_r^c–γ_r curve can be constructed to characterize the work hardening in the course of simultaneous torsion and extension.

4. Conclusions

Using the principle of plastic work connections between the shear and tensile strains and resolved shear strains are given for ⟨1 0 0⟩ and ⟨1 1 1⟩ textured and for nontextured polycrystals in the case of simultaneous shear and extension.

The application of the plastic work equation leads to deformation increments which fulfil the normality condition prescribed by the generalized flow law. The stress state and the deformation increments can be characterized by parallel vectors. In the case of simultaneous shear and extension the work hardening can be characterized by a single curve.

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